A comparison between micropolar elasticity and lattice structures

Department of Mechanical Engineering, Clemson University
Marcus Yoder, Ph.D. Student

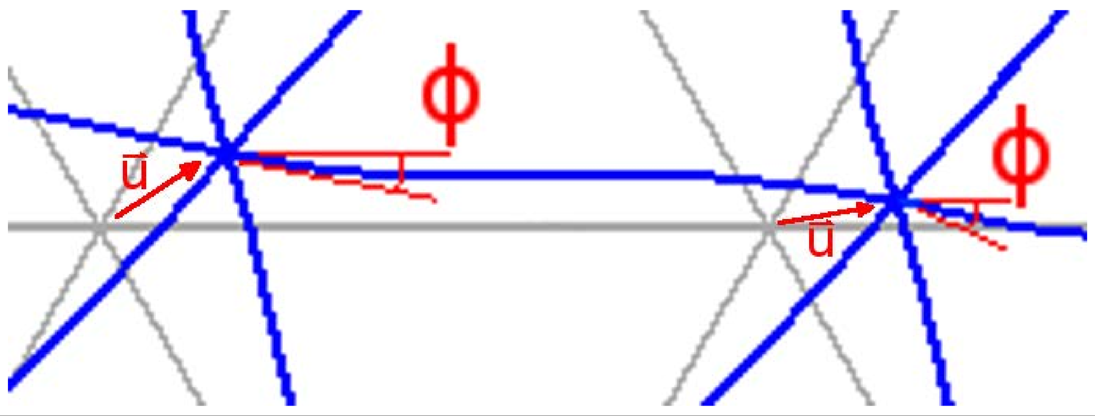
Advisors, Dr. Lonny Thompson, Associate Professor,
and Dr. Joshua Summers, Professor

Overview and Motivation

Micropolar elasticity theory is an extension of classical elasticity theory that incorporates a rotational degree of freedom. This rotational degree of freedom, ϕ, allows micropolar elasticity to better model porous materials, such as honeycombs and other periodic lattice structures.

Periodic lattice structures can be accurately modeled as a network of beams, therefore the entire displacement behavior of the lattice can be described by the displacement and rotation of the nodes. Micropolar elasticity's ϕ, is equivalent to the rotation of the nodes in the lattice.

Figure 1: Displacement and Rotation of node points in a portion of a triangular lattice. Reference configuration in gray. Deformed configuration in blue.



Constitutive Equations for strains and force stresses

Constitutive Equations & Strain Definition for couple stresses and curvature

Strain Definition for strains

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\\ \sigma_{21}\end{bmatrix} = \begin{bmatrix}\lambda+2\cdot\mu & \lambda & 0 & 0\\ \lambda & \lambda+2\cdot\mu & 0 & 0\\ 0 & 0 & \mu+\frac{\kappa}{2} & \mu-\frac{\kappa}{2}\\ 0 & 0 & \mu-\frac{\kappa}{2} & \mu+\frac{\kappa}{2}\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\\ \varepsilon_{21}\end{bmatrix}$$

$$\begin{bmatrix} m_{13} = \gamma\cdot k_{13} = \gamma\cdot\frac{\partial\phi_3}{\partial x_1}\\ m_{23} = \gamma\cdot k_{23} = \gamma\cdot\frac{\partial\phi_3}{\partial x_2}\end{bmatrix}$$

$$\begin{bmatrix}\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}\\ \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}\\ \varepsilon_{12} = \frac{\partial u_2}{\partial x_1} - \phi_3\\ \varepsilon_{21} = \frac{\partial u_1}{\partial x_2} + \phi_3\end{bmatrix}$$

Equations 1: Basic equations for planar isotropic micropolar elasticity.
σ_{ij} and ε_{ij} are the stress and strain on the i face in the j direction.
λ and μ, are Lame's constants. κ, and γ are material constants for micropolar elasticity.
m_{i3} and k_{i3} are the couple stress and curvature on the i face around the out of plane axis.

The partial derivatives of ϕ, curvature, are linked to an additional quantity called couple stress, in the same way that classical strain is related to force stress. The material properties of planar isotropic micropolar materials are fully characterized by 4 material constants. For simplicity, this poster restricts the discussion to planar isotropic lattice structures.

Various authors [1] have derived effective properties for lattices, or written tools to automate the process. Each of these authors did some validation to check that their derived material properties produce micropolar behavior that approximately matches the lattice behavior under ideal conditions. This research will investigate the how parameters like macro-size and boundary conditions affect the quality of that comparison.

Progress Made

This research is based on a pair of in-house Finite Element (FE) simulations. The codes simulates one of the lattice structures considered in [1] and an equivalent micropolar continuum using the material properties found in [1]. Both codes apply the same boundary conditions, have the same macro-size, and return the totals of the forces on the prescribed displacement face. An error metric is calculated based on the squared percent difference. Comparisons of the displacement are also available. Figure 2 shows the lattice types used in the code. Figure 3 shows a pair of simulations.

Reference:

[1] Kumar, R. S., and McDowell, D. L., 2004, "Generalized continuum modeling of 2-D periodic cellular solids," Int. J. Solids Struct., 41(26), pp. 7399–7422.

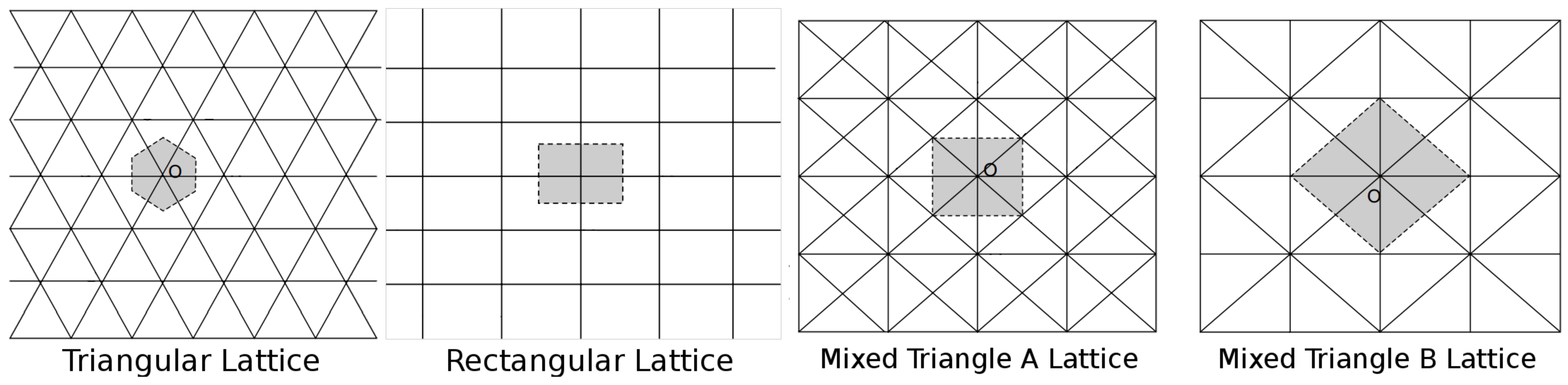


Figure 2: 4 types of lattices analysed by [1] and examined in this research.

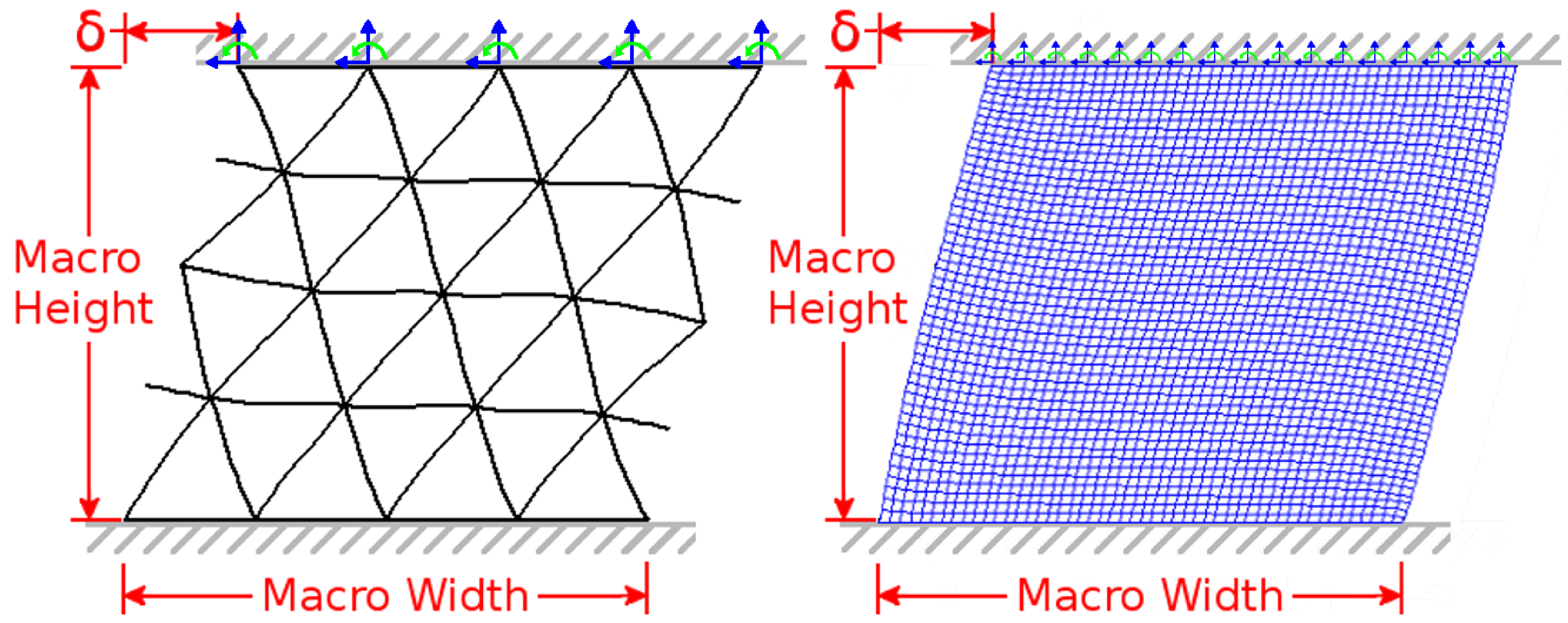


Figure 3: Simulations of a triangular lattice in shear (left) and its equivalent micropolar continuum (right).

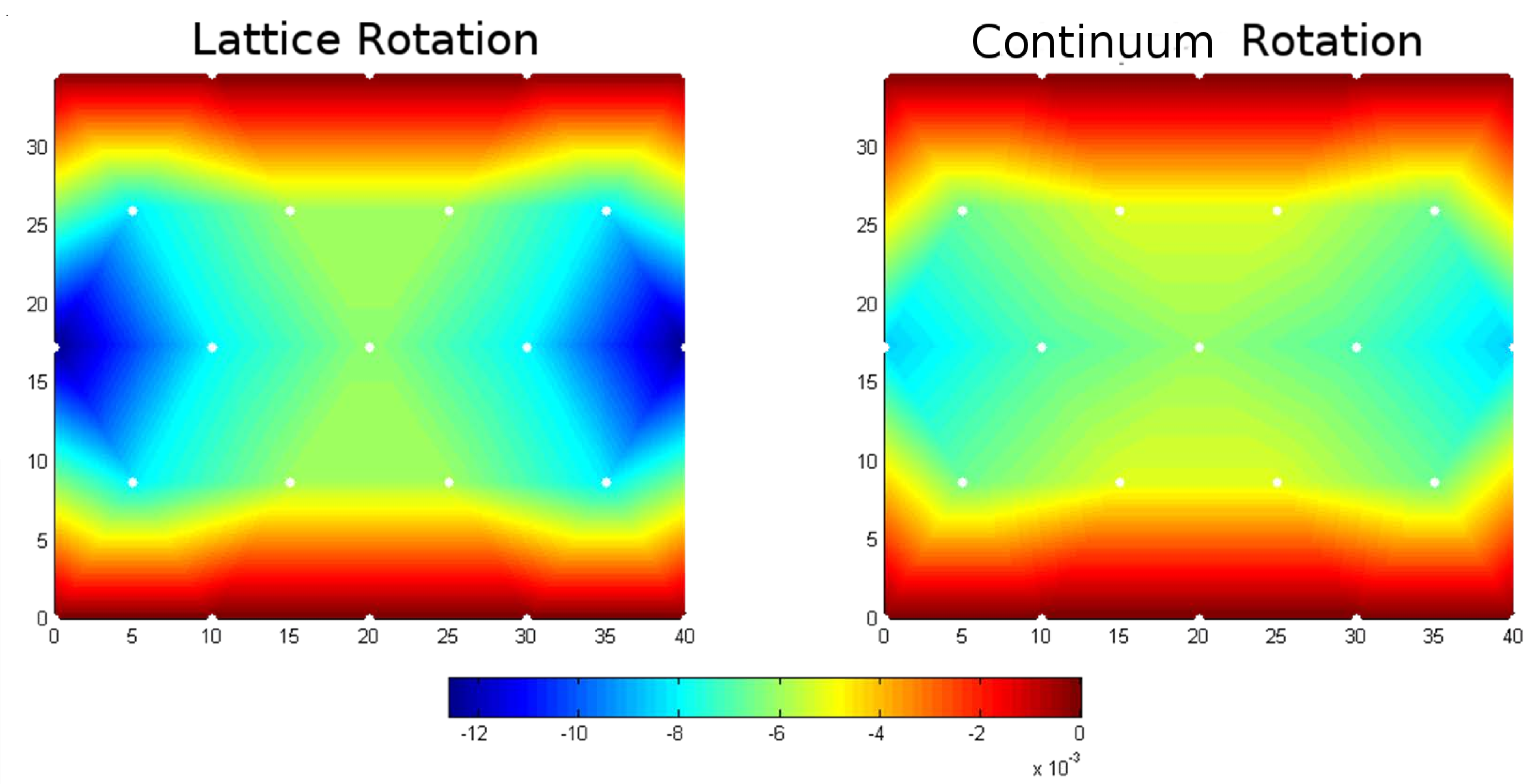


Figure 4: Micro-rotation of the lattices shown in Figure 3.

Early Investigations

In order to investigate whether the material properties from [1] give a smaller error metric than any other set of material properties, the tool was tied to an optimization routine to minimize the error, by varying the material properties. The material properties that gave the minimum error for all four lattices was different from the properties given by [1] by an order of magnitude.

This suggests that the comparison tool is not functioning properly.

Future work

In the short term, the tool needs to be checked for consistency and verify that every part of the code works.

In the long term, if micropolar elasticity theory can provide good comparisons, the theory can be extended to acoustic wave propagation and insulation. Lattice structures have the potential to combine good acoustic insulation with, high stiffness and high specific strength.

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For more information, please contact
Marcus Yoder at MarcusY@Clemson.edu